



$$D) \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

Series converges b/c Area converges

$$E) \sum_{n=1}^{\infty} \frac{1}{n^2 + 9}$$

$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2 + 9} dx = \lim_{b \rightarrow \infty} \left[\frac{1}{3} \arctan\left(\frac{x}{3}\right) \right]_1^b = \lim_{b \rightarrow \infty} \left[\frac{1}{3} \arctan\left(\frac{b}{3}\right) - \frac{1}{3} \arctan\left(\frac{1}{3}\right) \right]$$

$$\frac{1}{3} \left(\frac{\pi}{2} \right) - \frac{1}{3} \arctan\left(\frac{1}{3}\right)$$

Area Converges

P-Series Test

$$A) \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$p=2 > 1$ converges

$$B) \sum_{n=1}^{\infty} \frac{1}{n}$$

Harmonic Diverges $p=1 \leq 1$ diverges

$$C) \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$$

$p=1/3 \leq 1$ diverges

$$D) \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

$p=3/2 > 1$ convergence

$$E) \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$$

$p=5/2 > 1$ converges

$$F) \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$$

$p=1/2 \leq 1$ diverges

original terms
smaller than
comparison

Direct Comparison Test

A) $\sum_{n=1}^{\infty} \frac{1}{n^2+9}$ compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $p=2 > 1$
converges

$\sum \frac{1}{n^2+9}$ converges b/c $\frac{1}{n^2+9} < \frac{1}{n^2}$

$$\sum \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

$$\sum \frac{1}{n^2+9} = \frac{1}{10} + \frac{1}{13} + \frac{1}{18} + \frac{1}{25} + \dots$$

original terms
bigger than
comparison

B) $\sum_{n=4}^{\infty} \frac{n}{n^2-9}$ compare to $\sum_{n=4}^{\infty} \frac{1}{n}$ Harmonic
diverges

$\sum_{n=4}^{\infty} \frac{n}{n^2-9}$ diverges b/c $\frac{n}{n^2-9} > \frac{1}{n}$

$$\sum_{n=4}^{\infty} \frac{1}{n} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \dots$$

$$\sum_{n=4}^{\infty} \frac{n}{n^2-9} = \frac{4}{7} + \frac{5}{16} + \frac{6}{27} + \frac{7}{40} + \dots$$

C) $\sum_{n=1}^{\infty} \frac{n}{n^2+9}$ Compare to $\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic
Diverges

$$\sum \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

$$\sum \frac{n}{n^2+9} = \frac{1}{10} + \frac{2}{13} + \frac{3}{18} + \frac{4}{25} + \dots$$